REVIEWS

Hydrodynamics of Oceans and Atmospheres. By CARL ECKART. London: Pergamon Press, 1960. 290 pp. 63*s*.

From the title of this book, a prospective reader might expect a direct physical attack on the finite-amplitude turbulent convective circulations of the atmosphere and oceans, which would be intimately connected with observation and would suggest new types of physical measurement to decide such critical hypothetical elements as might occur in the theoretical development. Probably the word 'hydrodynamics' conveys some more restricted and formal meaning to the author than it does to the reviewers, because this book is principally a systematic study of wave motions of infinitesimal amplitude in an inviscid compressible stratified fluid on an infinite plane or on a sphere, with and without rotation. Starting with a concise and rather interesting discussion of the relevant thermodynamic concepts, the author formulates the basic problem of the book as the study of the hydrodynamic equations by the methods of perturbation theory. In this he takes the zero-order state to be one of no motion, which is regarded as given, and attention is thereafter essentially restricted to the first-order perturbations. Chapter III presents some discussion of steady solutions to the perturbation equations, and points out that the second-order equations cannot be completely neglected even in a first-order theory, since in some cases they imply relations among the first-order quantities which are not consequences of the first-order equations. Apparently the analogy of this with the process of 'suppression of secular terms' leads to these relations being called 'secular equations' by the author—a terminology which seems somewhat unfortunate because of the extensive use of this term as a synonym for 'eigenvalue equation'. Indeed, the widespread use of such special terms and symbols makes the book somewhat difficult to use for reference purposes, though it must be admitted that the author often has good reasons for his usage. When Chapter VIII is reached, a set of variables convenient for the study of waves has been introduced, the equations have been put into a standard form and their mathematical properties discussed briefly, and we are ready to attack some specific problems. The remainder of the book discusses such problems—basically the free waves of various systems with plane and spherical level surfaces, with and without rotation. Here we find a number of interesting things—a unified discussion of acoustic and gravity waves, a very thoroughgoing presentation and use of the ray theory, a clear discussion of the significance of the 'unlimited' (i.e. not square integrable) eigenfunctions in unbounded atmospheres, illustration of the applications of the WKB approximation, and some general results obtainable by the use of comparison and oscillation theorems. The principal limitation in this study of waves is that only the simplest types of boundary conditions have been considered—the upper and lower boundaries are always level surfaces, and the lateral boundaries are mostly not present at all; the case of vertical lateral boundaries is discussed only briefly in the simplest possible cases, without rotation. Thus edge waves and Kelvin waves are not mentioned. In addition, there is essentially no discussion of the excitation of waves by external influences, which is a pity since there are many important questions in this area which are accessible and of which a discussion within the author's general framework would have been most interesting. The only approach to such questions is in Chapter VI, 'General Theorems Concerning the Field Equations', where it is simply pointed out that in principle the solutions to the equations with forcing terms can be found once the eigen-solutions are known. But even here the emphasis on free waves has led the author to the assertion that for a finite volume of fluid the eigen-solutions are denumerable, and every regular (i.e. C^1) solution of the time-dependent first-order equations is expressible in the form

$$\mathbf{U} = \Sigma_n A_n(t)\, \mathbf{U}_n(x, y, z), \quad \text{etc.}$$

Counter-examples to this statement have already been given in Chapter III on 'Steady Motions'—the eigenvalue zero is always non-denumerably infinitely degenerate; even in its linearized version, hydrodynamics is not as simple as quantum mechanics. In this connexion, the author notes (p. 37) that with spherical level surfaces and rotation the only steady motions are the zonal geostrophic flows, i.e. the infinite degeneracy of the eigenvalue zero is partially split by the effects of rotation and spherical level surfaces; but his assertion that this effect of rotation has been overlooked in the past is unwarranted (see, for instance, Lamb's *Hydrodynamics*, §§206, 212, 223). In general, the properties of waves in rotating systems are much more widely known than is suggested by the limited bibliographic references; another example is the relation between Rossby waves and tidal oscillations of the second kind (mentioned on the dust jacket as an example of the unification brought about by the use of modern methods of atomic theory). This was pointed out by Haurwitz in 1940.

As is evident from the above sketch of the contents of the book, there are many parts of the hydrodynamics of oceans and atmospheres having considerable physical interest which are not discussed: the stability of waves on baroclinic currents in the atmosphere, the general circulation of the ocean, exchange processes across the sea–air interface, everything related to viscosity or turbulence, all problems related to heat transfer, such as the oceanic thermocline, all problems related to water vapour in the air, such as clouds, all problems related to radiation, such as the mean stability of the atmosphere. But the author states clearly at the beginning that such problems will not be discussed, and he can only be criticized for choosing an all-inclusive title. From the physical point of view, however, there is one fundamental objection to a basic feature of the perturbation-theory approach used. This is that the basic state (zero-order solution) may be regarded as given, and not subject to friction and heating. There are phenomena in the atmosphere and ocean for which this procedure is satisfactory, and for which the linearized perturbation equations then yield very useful information. Examples of such phenomena are tidal motions, refraction of sound and of surface waves, lee waves, tsunamis, and seiches. However, it is important to realize *that these are all phenomena which have little or no effect on the basic state.* A much more difficult problem is encountered in studying those phenomena in which the basic state is itself greatly influenced by the non-steady motions. Indeed, it is the explanation of the basic or average state that is frequently the most interesting and challenging aspect of the problems, and it is here that the fundamental physical questions of oceanographic and meteorological hydrodynamics, more often than not, lie. Under these conditions, it is certainly not permissible to disregard the effects of friction and heating on the zero-order solution. This is not to say that the perturbation technique is then worthless, but only that the mathematical difficulties become much more extreme than those treated elegantly by Dr Eckart, and that other techniques may be more productive.

Chapter III, on 'Steady Motions', if it is to be taken as some sort of a model of the real atmosphere, furnishes several examples of the dangers involved in applying the perturbation approach without a knowledge of the actual orders of magnitude involved. Friction is neglected even in the first-order equations, and the horizontal winds are therefore geostrophic. As a result, the author is forced to require that the horizontal pressure gradient vanish at the equator—in spite of observational evidence to the contrary. One of the more frustrating problems in meteorology is thereby denied existence. The author also uses the horizontal divergence of the geostrophic wind to deduce that air rises and is heated where the average surface pressure increases from west to east, and sinks and is cooled where the pressure decreases from west to east. This is not in agreement with observational evidence. Furthermore, the same relationship leads to the absurd result that the heating must vanish when averaged along any latitude circle! These difficulties are of course traceable to the author's choice of a basic state, and the neglect of friction, and should not be interpreted as a failure of the perturbation method *per se*. A much more fundamental error is the neglect of the now well-established *observational* evidence that the 'friction' and 'heating' which determine the average large-scale motions of the atmosphere are to a great extent a result of the non-steady motions. One questions then the physical relevance of a procedure which enables the steady-state solution to 'be considered independently of the components that change with time' (p. 22).

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Lectures on Fluid Mechanics. By S. GOLDSTEIN, with J. M. BURGERS. New York: Interscience, 1961. 309 pp. $ 6.60 or 50*s*.

From 23 June to 19 July 1957, the American Mathematical Society held a Summer Seminar in Applied Mathematics at Boulder, Colorado, to indoctrinate a general mathematical audience in four broad regions of applied mathematics. Each of the four groups of lectures has since appeared as a book, including the present volume on Fluid Mechanics, with 274 pages by S. Goldstein and 29 pages by J. M. Burgers.

One of the pleasures of reading a book of this kind is in seeing an author's personality shine through the printed page much as it must have done in the lecture room. This is not least so in Prof. Goldstein's preface, where he explains that he has attempted to offer only an enticing taste of fluid mechanics; that, clearly, no sort of completeness was possible; and that he has included, in the main, topics that have interested him. Incidentally, he gives later enough brief mention, with references, to the major branches of fluid mechanics which are not otherwise treated to avoid the well known dangers of innocent readers ignoring their existence!

The earlier chapters will give the connoisseur of Sydney Goldstein's Cambridge and Manchester lectures on hydrodynamics many a happy moment of recollection. To this classical material has been added an account of the elements of compressible flow, including boundary-layer aspects, as well as of the foundations of continuum magnetohydrodynamics.

The more advanced material includes a sketch of our knowledge of the exact two-dimensional incompressible flow past a semi-infinite plate. At a distance x behind the leading edge that is large compared with ν/U (where ν is kinematic viscosity, and U mainstream velocity), the flow is asymptotically determinate, as the Blasius solution within the boundary layer, merging outside it into the potential flow around the parabolic displacement-thickness contour. Early writers thought that higher approximations for large Ux/ν would also be determinate, and calculable by expansion in descending powers of Ux/ν. It now appears that the expansion must include logarithmic terms, and also arbitrary constants. The need for the latter is at once evident when we consider that the equations determining the asymptotic expansion would be unaffected by the presence of a bulbous rim along the leading edge, which, nevertheless, would cause an alteration to the boundary layer that would only gradually attenuate as Ux/ν increased. In fact, these constants can be determined only by joining the asymptotic solution on to a suitable numerical solution in the neighbourhood of the leading edge.

The book ends with some plasma dynamics. The calculation of electrostatic wave motions, derived by perturbing the collision-free Boltzmann equation for the electron velocity distribution about a Maxwellian distribution, is given, with some discussion of Landau damping. In addition, Dr Burgers makes a contribution that includes his calculation of the effects of a shock wave passing through fluid of large conductivity in the field of a magnetic dipole.

One's only regret, in a book like this aimed at giving a first taste of fluid mechanics, is the paucity of figures (seventeen altogether). Even though it may have been necessary to accept what it is not surprising to find the famous editor of *Modern Developments in Fluid Mechanics* calling 'one glaring and horrible omission—the omission of any comparison of theoretical with experimental results', nevertheless some diagrammatic representations of even calculated flow fields would have helped readers, unfamiliar with the subject's flavour, to appreciate that spicy component imparted to it by retinal reception of flow figuration.

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